Technical Comments

Comments on "A Re-Evaluation of Jet Damping"

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The "jet damping" effect on the axial spin of a slender solid-propellant rocket was the subject of a Note by Warner and Snyder.[1] The governing equation for their model was written from Thomson[2] in a slightly modified form, as

$$M_x = I_x\dot{\omega}_x + \dot{I}_x\omega_x + \omega_x \int_s \rho(r'^2_y + r'^2_x)(\mathbf{v}\cdot\mathbf{n})ds \quad (1)$$

We wish to note here that this equation is correct only in a particular case, that is, when the combustion gases maintain a constant rotational velocity along the exit plane which is equal to the spin rate of the rocket. This is the case only if the jet viscosity is very high or if the exit is made of a cluster of an infinite number of infinitesimally small nozzles (a "mesh" nozzle). However, these conditions are not met in most practical cases.

The equation of motion of a spinning rocket may be written as

$$M_x = I_x\dot{\omega}_x + \dot{I}_x\omega_x + \int_s \omega_\rho r^2\rho(\mathbf{v}\cdot\mathbf{n})ds \quad (2)$$

ω_ρ is the rotational velocity of elemental mass around the axis of symmetry, x. The two last terms of the right-hand side of Eq. (2) represent the rate at which angular momentum (relative to the spinning rocket) leaves the system.

It is interesting to note two particular cases besides that of a "mesh" nozzle:

1) Frictionless gas, one nozzle, and tubular solid-fuel grain (body of revolution). In this case, the angular momentum of the combustion gases equals the angular momentum of the fuel since there are no shearing forces between the gas and the rocket. Consequently, there is no transfer of angular momentum between the gases and the rest of the system; the last two terms of the right-hand side of Eq. (2) cancel each other, and the angular velocity of the rocket remains constant if the external moment M_x is zero.

2) Frictionless gas, one nozzle, and starlike cross section of the fuel grain. It may be shown that the flow of the gases from the outer corners of the grain to the central section (in the usual case when the throat diameter is smaller than the outer diameter of the grain channel) causes slight angular acceleration of the rocket.

In general, for real fluids and large (probably multiple) nozzle we have to evaluate ω_ρ at each point of the exit plane before attempting to solve (2).

References

[1] Warner, G. G. and Snyder, V. W., "A Re-Evaluation of Jet Damping," *Journal of Spacecraft and Rockets*, Vol. 5, No. 3, March 1968, pp. 364–366.

[2] Thomson, W. T., *Introduction to Space Dynamics*, Wiley, New York, 1961, pp. 221–227.

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Comments on "A Re-Evalution of Jet Damping"

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In a recent Technical Note[1] Warner and Snyder set out to re-examine the effect of variable radius of gyration on jet damping in spin. As a result of extensive parametric analysis, which is essentially sound and appears to be a logical extension of former work on the subject,[2] they conclude that the spin rate of solid-fuel space rockets is subject to gross changes due to jet damping, which may, further, be strongly affected by varying radius of gyration. Noting that these results may not be supportable by experimental evidence, Warner and Snyder observe that answers from any analytical investigation are as good as the model analyzed. Yet we note that their model represents a reasonable approximation of a typical solid-fuel space rocket with a single nozzle.

The discrepancy between results reported in Ref. 1 and actual flight test data follows directly from the fact that the expression for the spin component of the angular momentum transfer rate used in the analysis is not consistent with the model. This expression is written by the authors in the form $\omega_x\int\rho(r_y^2 + r_z^2)(\mathbf{v}\cdot\mathbf{n})ds$ and subsequently reduced to $-(\dot{m}r^2/2)\omega_x$, where ρ is the exhaust gas density, $\mathbf{v}'$ is the velocity of gases relative to exit plane, $\mathbf{n}$ is the unit normal to exit plane, r is the exit radius, and ω is the spin velocity of the rocket; r_y and r_z, not mentioned by the authors, are obviously the components of the radial distances of individual mass particles from the spin axis. Explicit in these expressions is an assumption that spin velocity of the exhaust gases is always equal to that of the rocket.

For this to be true in view of the aforementioned line integral, the exit of the nozzle would have to be equipped with a honeycomb-like structure with minute openings and of sufficient length to impart to each streamline tangential velocity consistent with the spin velocity of the rocket. The model considered in Ref. 1 (single nozzle, radial burning) has no mechanism available to force the fluid matter to spin with velocity equal to that of the rocket. Assumption of radial burning implies absence of Coriolis forces in transverse planes. From this fact and from the fact that viscous effects are negligible, it follows that there can be no interaction in spin between the rigid rocket and burned gases, i.e., there can be no jet damping in spin. The spin velocity of the rocket will remain constant at all times. For the same reasons, the angular spin momentum per unit mass of burned matter at the exit is the same as it was in the fuel core prior to burning. This implies that the spin velocity of the exhaust gases at the exit will, in general, be different from that of the rocket because the radius gyration of the exit and that of the burning surface about the axis of symmetry of the rocket are (in general) different. As burning proceeds and the second moment of the burning fuel surface about the spin axis increases, the rate of angular spin momentum transfer out of the system increases and so does the spin velocity of the exhaust. All these phenomena, however,

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have no bearing upon the spin of the rocket, which as stated before, remains constant.

The authors begin their analysis by stating that "the governing equations for the model can be written in a slightly modified form from Thomson."[2] Here lie the origins of misunderstanding. Thomson's model includes a cluster of nozzles, and in writing $\Sigma_i \dot{m}_i r_i^2$ he performs a summation of all the jets issuing from separate nozzles—a very different proposition from summing all the streamlines over a single nozzle, as in Ref. 1. Thus, in the case of Thomson's model, the rate of angular momentum loss along the axis of symmetry is indeed proportional to the spin velocity of the rocket, at least to the first degree of approximation. Variations in spin velocity of the rocket shown in Thomson's example are amenable to logical interpretation by assuming that all radially located nozzles are fed from a common fuel tank (or fuel core, if one insists upon interpreting his results in terms of a solid-fuel rocket) coaxial with the spin axis of the rocket. In such a case radial routing of the fuel and/or gases along some channels would provide the mechanism for production of Coriolis forces necessary for spin changes of the rocket.

Although data from actual flights do not bear out the phenomena reported in Ref. 1, small variations in the spin velocity of thrusting space rockets are commonly observed.[3] These are usually attributed to small jet damping in spin, which may result from complex phenomena associated with more elaborate geometries of the fuel core. The effect of a small spin torque that may be present due to combined thrust eccentricity and misalignment is negligible. Another source of spin velocity variations, which appears to have escaped attention so far, and yet is likely to be relatively large, exists in the form of a body-fixed transverse moment due to thrust. When a uniformly spinning, perfectly symmetric rocket is subjected to a body-fixed transverse moment due to thrust, the spin axis of the rocket originally coincident with the axis of symmetry tilts through an angle $\beta = -M/(I_x - I_y)n^2$, where M is the moment, n is the spin velocity of the rocket, and I_x and I_y are the roll and pitch moments of inertia, respectively. Since β is small and M is a purely transverse moment, it follows that the spin momentum about the new axis will be equal to the original spin momentum about the axis of symmetry. It is rather evident, therefore, that this rotational shift in the spin axis will be accompanied by a decrease in the spin velocity. As burning continues, M will increase and $(I_x - I_y)$ will decrease the two effects, resulting in a large increase of β; for a typical rocket, this increase of β is of an order of ten. As a consequence, a noticeable gradual drop in the spin rate during the later stages of burning may be produced, to be followed by a sharp increase in the spin rate immediately after burnout when the spin axis returns to the axis of symmetry. One should also note that misalignments and offsets (between the nozzle and fuel core, nozzle and spin axis, etc.) associated with formation and presence of moment due to thrust create conditions favorable for production of small Coriolis forces in transverse planes and small jet damping in spin. Postflight data from spin-stabilized solid-fuel rockets, based upon signal strength fluctuations, which this writer has examined, appear to support the foregoing, somewhat intuitive, deductions. This writer hopes that these deductions are sufficiently interesting (or provoking) to prompt other workers to a critical formal investigation of the subject.

References

[1] Warner, G. G. and Snyder, V. M., "A Re-Evaluation of Jet Damping," *Journal of Spacecraft and Rockets*, Vol. 5, No. 3, March 1968, pp. 364–366.

[2] Thomson, W. T., *Introduction to Space Dynamics*, Wiley, New York, 1961, pp. 221–227.

[3] Thomson, W. T. and Reiter, G. S., "Jet Damping of a Solid Rocket: Theory and Flight Results," *AIAA Journal*, Vol. 3, No. 3, March 1965, pp. 413–417.

Reply by Authors to P. Katz and to T. Papis

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THE comments are concerned with the basic assumption that the gases in the exit plane are rotating with the same angular velocity as the body. In the real vehicle this assumption may not be correct, but an accurate time history of the gas particles is difficult to obtain. The analysis could be modified as indicated by Katz if the rotational velocity were known a priori. It was indicated in both comments that no jet damping would be present for a frictionless gas. The authors' analysis[1] would give an upper bound to the effect of jet damping.

The mechanism for the transfer of angular momentum between the burned fuel and the vehicle is very complex. For the analysis presented by Warner and Snyder, the needed mechanism could be furnished best by a liquid-propellant rocket. The injector plate of the rocket engine would impart to the fuel some of the angular velocity required for the momentum transfer. A solid-propellant rocket is much easier to model mathematically than the liquid rocket but would rely mainly on the viscous effect of the gas.

Papis discussed other effects which could change the spin of the vehicle, and these effects may be greater than the jet damping. The authors are pleased with these constructive comments, for it is through discussions of this nature that knowledge is disseminated.

References

[1] Warner, G. G. and Snyder, V. W., "A Re-Evaluation of Jet Damping," *Journal of Spacecraft and Rockets*, Vol. 5, No. 3, March 1968, pp. 364–366.

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Comment on "Shock-Wave Shapes around Spherical- and Cylindrical-Nosed Bodies"

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A KNOWLEDGE of shock-wave shape and detachment distance around hypersonic blunt bodies is important for estimating shock-wave interference effects on winged- or finned-body missile configurations, for calculating re-entry body convective heating in the presence of large inviscid shock-layer entropy gradients, and for predicting radiative heat transfer from the high-temperature shock layers around manned superorbital re-entry vehicles (to mention just a few examples). Such knowledge can be obtained from detailed, numerical blunt-body flowfield analyses that give solutions

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